

M.Sc Mathematics (Sem -III)

MM 561 TOPOLOGY

Pass percentage: 35%
Time allowed : 3 hours.

External Evaluation: 70
Internal Evaluation: 30

Course Outcomes:

CO1: Can differentiate between finite, countable, uncountable sets and understand the concept of open-sets, closed set, interior and exterior points.

CO2: Can understand the topological properties like compactness, connectedness and the countability axioms and find their numerous uses in the course.

CO3: The concepts of basis and sub-basis of a space, of interior and closure set the stage for the most general study of continuity.

CO4: Enables the student to understand the special characters of the metric spaces as an important special case of a topological space.

INSTRUCTIONS FOR THE PAPER-SETTER

The question paper will consist of three sections: A, B and C. Sections A and B will have four questions each from the respective sections of the syllabus. Section C will consist of one compulsory question having ten short questions covering the entire syllabus uniformly. Each question in sections A and B will be of 10 marks each and section C will be of 30 marks.

INSTRUCTIONS FOR THE CANDIDATES

Candidates are required to attempt five questions in all selecting two questions from each sections A and B and compulsory question of section C.

SECTION A

Cardinals: Equipotent sets, Countable and Uncountable sets, Cardinal Numbers and their Arithmetic, Bernstein's Theorem and the Continuum Hypothesis.

Topological Spaces: Definition and examples, Euclidean spaces as topological spaces, Basis for a given topology, Topologizing of Sets; Sub-basis, Equivalent Basis.

Elementary Concepts: Closure, Interior, Frontier and Dense Sets, Topologizing with pre-assigned elementary operations. Relativization, Subspaces.

Maps and Product Spaces: Continuous Maps, Restriction of Domain and Range, Characterization of Continuity, Continuity at a point, Piecewise definition of Maps and Neighbourhood finite families. Open Maps and Closed Maps, Homeomorphisms and Embeddings.

SECTION B

Cartesian Product Topology. Elementary Concepts in Product Spaces, Continuity of Maps in Product Spaces and Slices in Cartesian Products.

Connectedness: Connectedness and its characterizations, Continuous image of connected sets, Connectedness of Product Spaces. Applications to Euclidean spaces. Components, Local Connectedness and Components, Product of Locally Connected Spaces. Path Connectedness.

Compactness and Countability: Compactness and Countable Compactness, Local Compactness, One-point Compactification, T_0 , T_1 , and T_2 spaces, T_2 spaces and Sequences and Hausdorffness of One-Point Compactification.

Axioms of Countability and Separability, Equivalence of Second axiom, Separable and Lindelof in Metric Spaces, Equivalence of Compact and Countably Compact Sets in Metric Spaces.

Books Recommended

1. W.J. Pervin Foundations of General Topology, New York, Academic Press, Ch. 2 (Sections 2.1, 2.2), Section 4.2, and Ch 5 (Sec 5.1 to 5.3).
2. James Dugundji: TOPOLOGY. Allyn and Bacon. Relevant Portions from Ch.III (excluding Sec 6)
3. J. Kelley: Topology. Graduate Texts in Mathematics 27. Springer.

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MM-562(FUNCTIONAL ANALYSIS)

Duration: 3 Hrs.

Max. Marks: 100

Internal Assessment: 30

External Examination: 70

Course Learning Outcomes:

On completion of this course, the students will be able to:

CO-1 Understand the concept of norm and its completeness.

CO-2 Study the concept of Hahn-Banach Theorem, Open mapping theorem, Closed graphs.

CO-3 analyse the difference between Banach Spaces and Hilbert Spaces. Also check the totality of orthonormal sets.

CO-4 Understand the basic concepts of operators and its classification.

INSTRUCTIONS FOR THE PAPER-SETTER

The question paper will consist of five sections: A, B and C. Sections A and B will have four questions each from the respective sections of the syllabus. Sections C will consist of one compulsory question having ten short answer covering the entire syllabus uniformly. The weightage of section A and B will be 30% and that of section C will be 40%.

INSTRUCTIONS FOR THE CANDIDATES

Candidates are required to attempt five questions in all selecting two question from each sections A and B and compulsory question of section C.

SECTION-A

Normed Linear spaces, Banach spaces, Examples of Banach spaces and subspaces. Continuity of Linear maps, Equivalent norms. Normed spaces of bounded linear maps. Bounded Linear functional. Hahn-Banach theorem in Linear Spaces and its applications.

Hahn-Banach theorem in normed linear spaces and its applications. Uniform boundedness principle, Open mapping theorem, Projections on Banach spaces, Closed graph theorem.

SECTION-B

The conjugate of an operator. Dual spaces of l_p and $C[a,b]$, Reflexivity. Hilbert spaces, examples, Orthogonality, Orthonormal sets, Bessel's inequality, Parseval's theorem. The conjugate space of a Hilbert spaces. Adjoint operators, Self-adjoint operators, Normal and unitary operators. Projection operators. Spectrum of an operator, Spectral Theorem, Banach Fixed Point Theorem, Brower's Fixed Point Theorem. Schauder Fixed Point Theorem, Picards Theorem. Applications of Fixed point theorem in differential equations and integral equations.

Books Recommended

1. G.F.Simmons: Introduction to Toplogy and modern Analysis, Chapters IX, X, XII and appendix one.

Ashish

Payal

Yog

Pawan Lal

M.Sc Mathematics (Sem -III)
MM 563 FIELD THEORY

Pass percentage: 35%
Time allowed : 3 hours.

External Evaluation: 70
Internal Evaluation: 30

Course objective : The objective of the course is to help the students acquire skills to check irreducibility of polynomials by different tests and to provide framework for Galois Theory which helps in solvability of quintic equations.

Course learning outcomes: On completion of course, the student will be able to

CO-I : Understand irreducibility of polynomials using various irreducibility tests.

CO-II : Understand field extension particularly Normal extensions and Seperable extensions.

CO-III : Understand the concept of splitting field of a polynomial and its evaluation.

CO-IV : Understand Galois Theory and its applications particularly solving a quintic equation.

CO-V : Understand Fundamental Theorem of Algebra.

INSTRUCTIONS FOR THE PAPER-SETTER/EXAMINER

The question paper will consist of three sections A, B and C. Sections A and B will have four questions each from the respective sections of the syllabus and Section C will consist of one compulsory question having ten short answer type questions covering the entire syllabus uniformly. Each question in Sections A and B will be of 10 marks and Section C will be of 30 marks.

INSTRUCTIONS FOR THE CANDIDATES

Candidates are required to attempt five questions in all selecting two questions from each of the Sections A and B and compulsory question of Section C.

SECTION:A

Fields ,examples, algebraic and Transcendental elements, irreducible elements, Guass Lemma, Eisenstein's Criterion, adjunction of roots, Kroneckor's theorem, algebraic extensions, algebraically closed fields, Splitting fields, Normal extensions, multiple roots, finite fields, seperable extensions, perfect extensions, perfect fields, primitive elements, Langrange's theorem on primitive element.

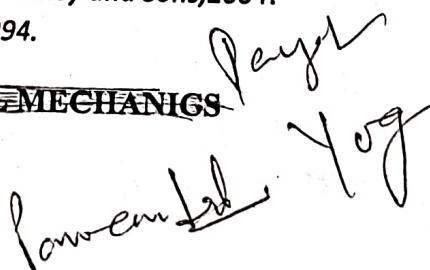
SECTION :B

Automorphism groups and fixed fields, Galois extensions, Fundamental theorem of Galois theory, Fundamental theorem of algebra, roots of unity and Cyclotomic polynomials, Cyclic extensions, Polynomial solvable by radicals, Cyclotomic extension, quintic equation and solvability by radicals.

Books Recommended:

1. P.B. Bhattacharya, S.K Jain ,S.R.Nagpal: *Basic Abstract Algebra*, 2nd Edition, Cambridge University Press, 2002.
2. D.S. Dummit, Richard M Foote: *Abstract Algebra*, John Wiley and Sons, 2004.
3. M. Artin: *Algebra*, Prentice Hall of India, New Delhi, 1994.

~~MM 564-CLASSICAL MECHANICS~~



MM 564-CLASSICAL MECHANICS

Pass percentage: 35%
Time allowed : 3 hours.

External Evaluation: 70
Internal Evaluation: 30

Objective: The subject of Classical Mechanics is a perfect example of the power of mathematics to solve real physical problems and this course introduces the students to the Lagrangian version of Classical Mechanics which is indispensable for any study of Quantum Mechanical Methods

INSTRUCTIONS FOR THE PAPER-SETTER

The question paper will consist of three sections A, B and C. Sections A and B will have four questions each from the respective sections of the syllabus and Section C will consist of one compulsory question having ten short answer type questions covering the entire syllabus uniformly. Each question in Sections A and B will be of 10 marks and Section C will be of 30 marks.

INSTRUCTIONS FOR THE CANDIDATES

Candidates are required to attempt five questions in all selecting two questions from each of the Section A and B and compulsory question of Section C.

SECTION-A

Basic Principles: Mechanics of a Particle and a System of Particles, Constraints, Generalized Coordinates, Holonomic and Non-Holonomic Constraints. D'Alembert's Principle and Lagrange's Equations, Velocity Dependent Potentials and the Dissipation Function, Simple Applications of the Lagrangian formulation. **Variational Principles and Lagrange's Equations:** Hamilton's Principle, Derivation of Lagrange's Equations from Hamilton's Principle, Extension of Hamilton's Principle to Non-Holonomic Systems. **Conservation Theorems and Symmetry Properties:** Cyclic Coordinates, Canonical Momentum and its Conservation, The Generalized Force, and Angular Momentum Conservation Theorem. **The Two-Body Central Force Problem:** Reduction to the Equivalent One-Body Problem, The Equation of Motion, The Equivalent One Dimensional Problem and the Classification of Orbits, The Virial Theorem, Conditions for Closed Orbits, Bertrand's Theorem.

SECTION-B

The Kepler Problem: Inverse Square Law of Force, The Motion in Time in the Kepler Problem, Kepler's Laws, Kepler's Equation, The Laplace-Runge-Lenz Vector.

Scattering in a Central Force Field: Cross Section of Scattering, Rutherford Scattering Cross Section, Total Scattering Cross Section, Transformation of the Scattering Problem to Laboratory Coordinates. The Independent Coordinates of Rigid Body, The Transformation Matrix, The Euler Angles, The Cayley-Klein Parameters and Related Quantities, Euler's Theorem on the Motion of Rigid Bodies, Finite Rotations, Infinitesimal Rotations, The Coriolis Force.

Pedagogy: The instructor should lay emphasis on those techniques which naturally lend themselves to their quantum mechanical interpretations to enable the student to more naturally transform from the classical to the quantum.

RECOMMENDED BOOK

Herbert Goldstein: Classical Mechanics

Handwritten signatures:
Cousins
Payal
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Pawankumar

MM 565: NUMERICAL ANALYSIS

M.Sc Mathematics (Sem -III)

Pass percentage: 35%
Time allowed : 3 hours.

External Evaluation: 70
Internal Evaluation: 30

INSTRUCTIONS FOR THE PAPER-SETTER

The question paper will consist of three sections: A, B and C. Sections A and B will have four questions each from the respective sections of the syllabus. Sections C will consist of one compulsory question having ten short answer covering the entire syllabus uniformly. The weightage of section A and B will be 30% and that of section C will be 40%. Use of scientific calculator is allowed.

INSTRUCTIONS FOR THE CANDIDATES

Candidates are required to attempt five questions in all selecting two question from each sections A and B and compulsory question of section C. Use of non programmable scientific calculator is allowed.

SECTION-A

Solution of Differential Equations: Taylor's series, Euler's method, Improved Euler method, Modified Euler method, and Runge-Kutta methods (upto fourth order), Predictor Corrector methods. Stability and convergence of Runge-Kutta and Predictor Corrector Methods. Parabolic Equation: Explicit and Implicit schemes for solution of one dimensional equations, Crank-Nicolson, Du fort and Frankel schemes for one dimension equations. Discussion of their compatibility, stability and convergence Peaceman-Rachford A.D.I. scheme for two dimensional equations.

SECTION-B

Elliptic Equation: Finite difference replacement and reduction to block tridiagonal form and its solution; Dirichlet and Neumann boundary conditions. Treatment of curved boundaries; Solution by A.D.I. method.

Hyperbolic equations: Solution by finite difference methods on rectangular and characteristics grids and their stability. Approximate methods: Methods of weighted residual, collocation, Least-squares and Galerkin's methods. Variational formulation of a given boundary value problem, Ritz method. Simple examples from ODE and PDE.

BOOKS RECOMMENDED

1. Smith, G D, Numerical solution of partial differential equations, Oxford Univ. Press (1982).
2. R.S. Gupta, Elements of Numerical Analysis, Macmillan India Ltd., 2009. 3. Mitchell, A. R., Computational methods in partial differential equations, John Wiley (1975).
3. Froberg, C. E., Introduction to Numerical Analysis, Addison-Wesley, Reading, Mass (1969).

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